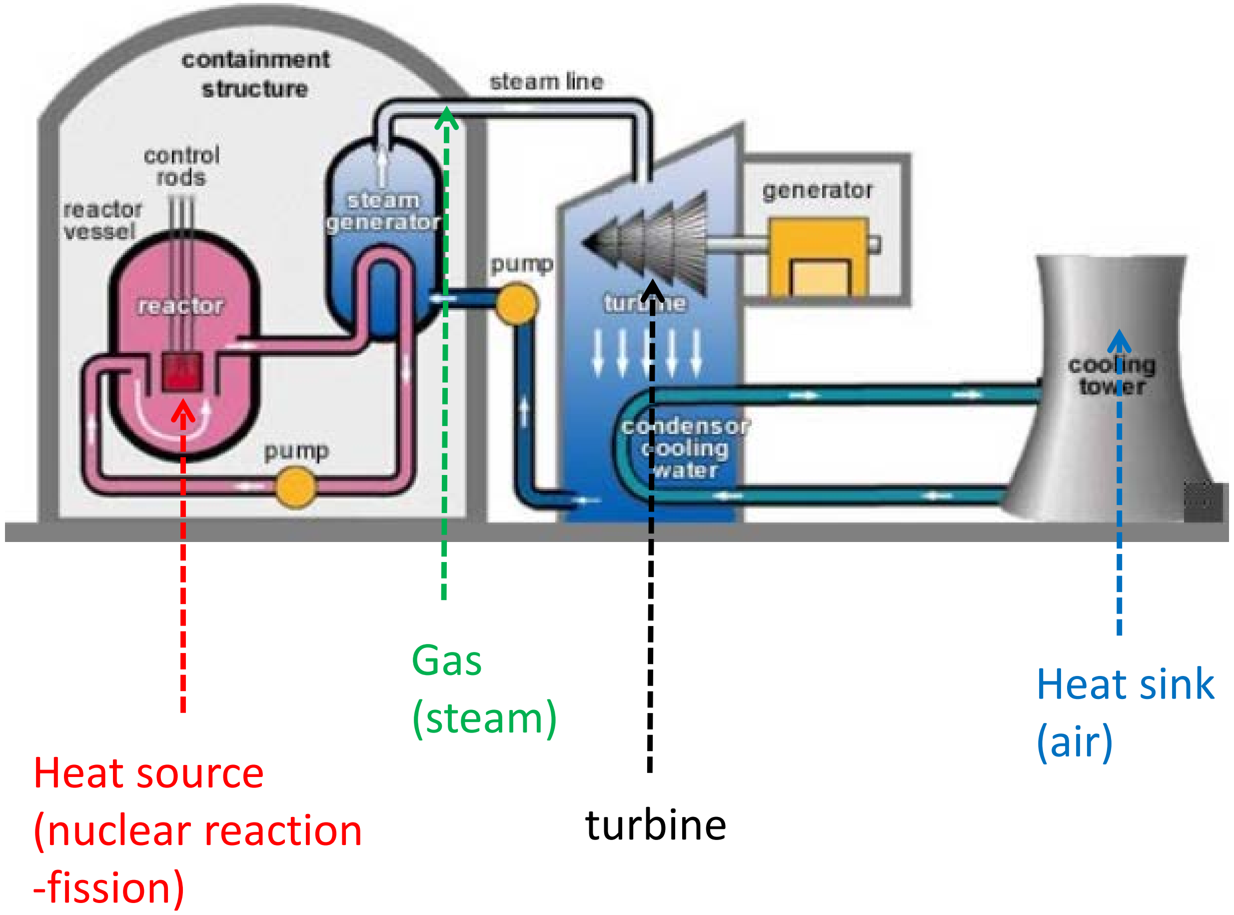
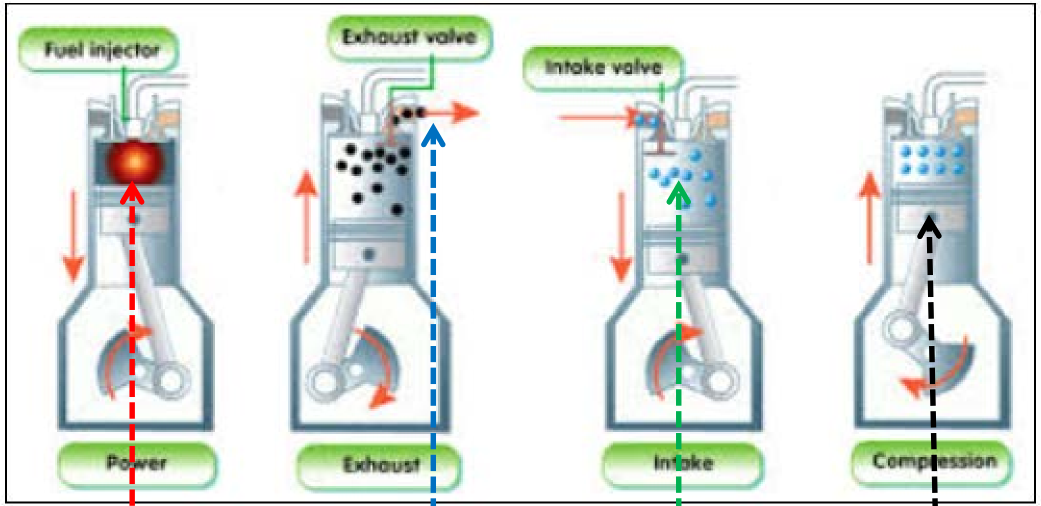
F.7 Thermodynamic Processes

The fundamentals of thermodynamics are now established. And next we can turn to applications: engines, and refrigerators. Either one usually consists of the following four elements:



- Heat source raises temperature/pressure of gas
- Gas does work on the piston/turbine pushing it up/rotating it.
- Heat sink lowers temperature/pressure of gas
- Cycle repeats.

F.7 Thermodynamic Processes



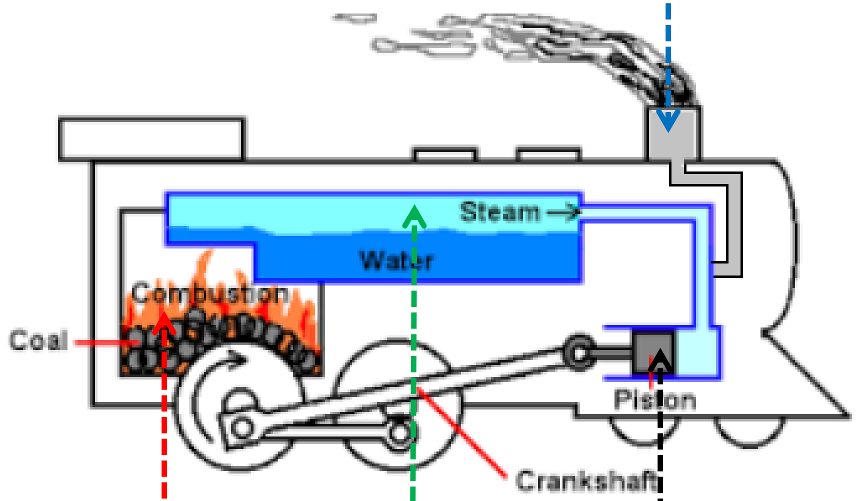
Heat source (chemical reaction – combustion)

Heat sink (air)

Gas (air/fuel mixture)

piston

Heat sink (air)



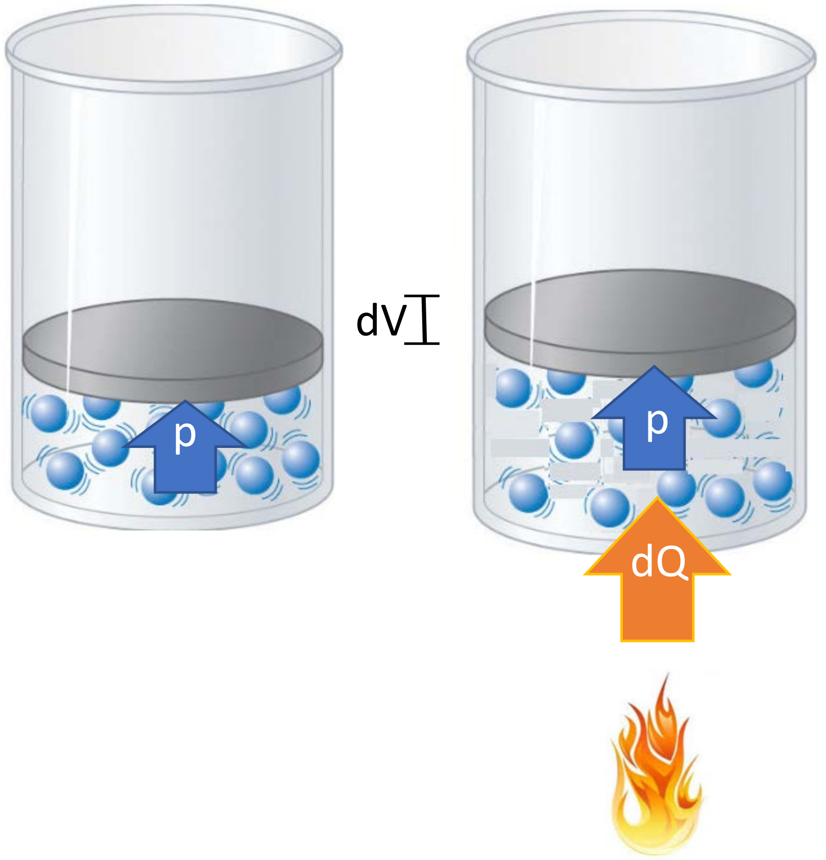
Heat source (chemical reaction -combustion)

Gas (steam)

piston

F.7 Thermodynamic Processes

It is critical to know what the pressure the gas will exert on the piston, when the gas is at a given temperature and volume. And we can get a formula for p as long as we know the two main properties of the gas: E(T,V,N), and S(T,V,N). This is how it works:



$$1^{st}\text{ law}: \; dW + dQ = dE_{gas}$$ This is differential form of 1st law

$$-dW_{gas} + dQ = dE_{gas}$$ Work done on gas is equal/opposite work done by gas

$$-pdV + dQ = dE_{gas}$$ Differential work is Fdx = pdV

$$2^{nd}\text{ law}: \frac{dQ}{T} + dS_{\text{int.}} = dS_{gas}$$ This is differential form of 2nd law

$$\frac{dQ}{T} + 0 = dS_{gas}$$ Assume gas is always in equilibrium so $dS_{\text{int.}} = 0$

$$dQ = TdS_{gas}$$

Now plug 2nd law into 1st law and solve for p:

$$-pdV + TdS = dE_{gas}$$

$$-p + T\frac{dS}{dV} = \frac{dE_{gas}}{dV}$$

$$\boxed{p = T\frac{dS_{gas}}{dV} - \frac{dE_{gas}}{dV}}$$

F.7 Thermodynamic Processes

So let's apply this to an **ideal gas** and see what we get:

$$E_{gas} = \frac{f}{2} N k_B T$$

$$S_{gas} = N k_B \ln\left(\frac{T^{f/2} V}{\Phi N}\right)$$

$$\longrightarrow$$

$$\begin{aligned} p &= T\frac{dS_{gas}}{dV} - \frac{dE_{gas}}{dV} \\ &= T\frac{d}{dV}\left[N k_B \ln\left(\frac{T^{f/2} V}{\Phi N}\right)\right] - \frac{d}{dV}\left[\frac{f}{2} N k_B T\right] \\ &= T N k_B \frac{1}{V} - 0 \end{aligned}$$

$$\longrightarrow \quad \boxed{p = \frac{N k_B T}{V}}$$

We could apply this formula to the **Van der Waals gas** too:

$$E_{gas} = \frac{f}{2} N k_B T - \frac{aN^2}{V}$$

$$S_{gas} = N k_B \ln\left(\frac{T^{f/2} (V - bN)}{\Phi N}\right)$$

$$\longrightarrow$$

$$\begin{aligned} p &= T\frac{dS_{gas}}{dV} - \frac{dE_{gas}}{dV} \\ &= T\frac{d}{dV}\left[N k_B \ln\left(\frac{T^{f/2} (V - bN)}{\Phi N}\right)\right] - \frac{d}{dV}\left[\frac{f}{2} N k_B T - \frac{aN^2}{V}\right] \\ &= T N k_B \frac{1}{V - bN} - \frac{aN^2}{V^2} \end{aligned}$$

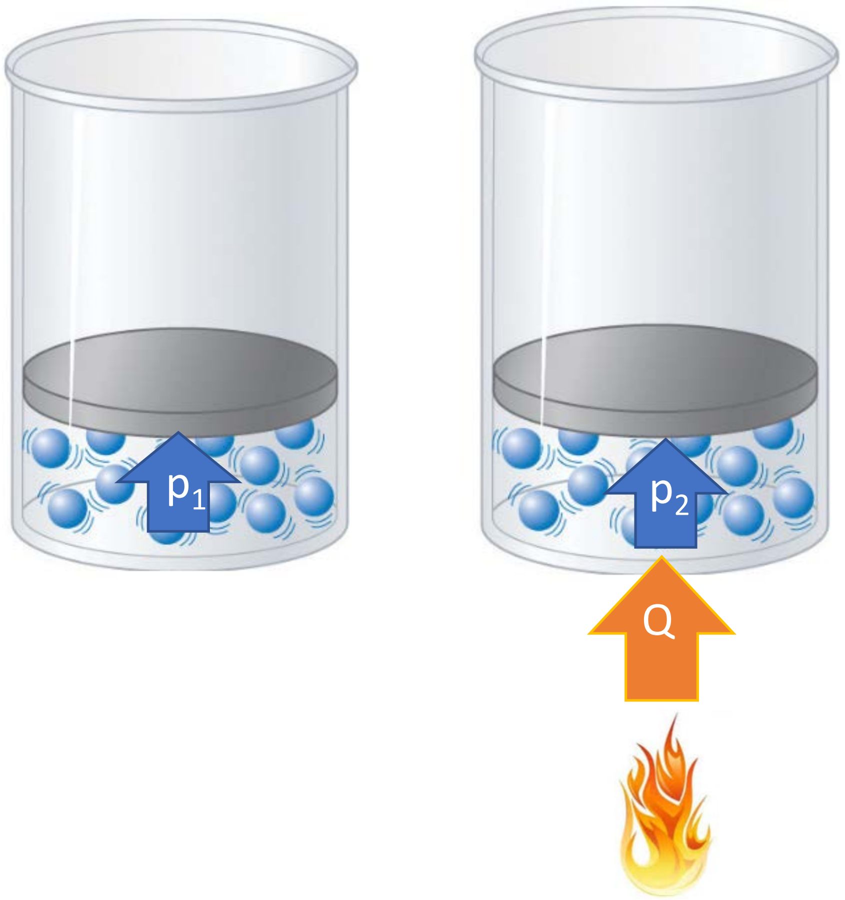
$$\longrightarrow \quad \boxed{p = \frac{N k_B T}{V - bN} - \frac{aN^2}{V^2}}$$

Could also apply to our **ideal solid** (formula doesn't *only* apply to gasses), but I let you do that ☺.

F.7 Thermodynamic Processes

By controlling the rate at which heat is added to the gas (or whether it is added at all), we can control how the pressure of the gas changes as it does work on the piston/turbine. And it turns out that controlling the pressure the gas exerts on the piston/turbine is crucial to an engine's efficiency. There are four main pressure processes that are used in engines.

1. Isometric process



Isometric means constant volume. Volume can be constant if....

- latch physically restrains piston from moving, or
- heat is added so *quickly* that the gas has little time to expand and so volume is *practically* constant.

What is W_{gas}?

$$W_{gas} = \int_{V_1}^{V_2} pdV = 0 \text{ (because V doesn't change)} \longrightarrow \boxed{W_{gas} = 0}$$

What is Q?

$$-W_{gas} + Q = \Delta E$$

$$Q = \Delta E + W_{gas}$$

$$= \frac{f}{2} Nk_B \Delta T$$

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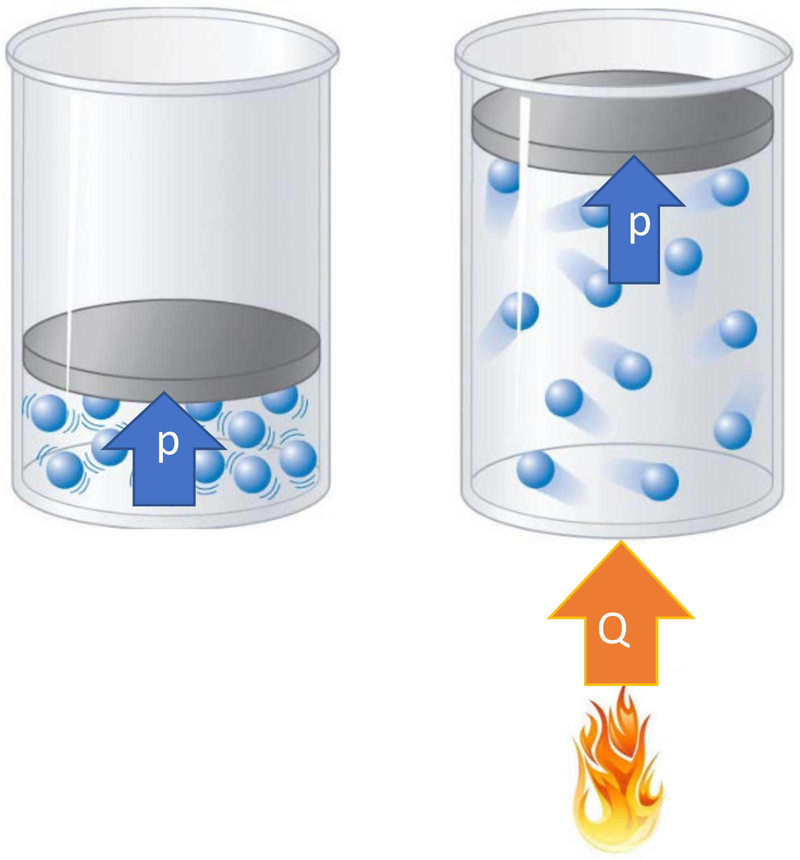
2. Isobaric process



Isobaric means constant pressure. pressure can be constant if….

- So as gas's volume increases, both its temperature and pressure will drop. Heat must be added at rate sufficient to make temperature *increase*, to compensate for the volume drop, and keep the pressure constant.

What is W_{gas}?

$$W_{gas} = \int_{V_1}^{V_2} p dV = p\int_{V_1}^{V_2} dV = p(V_2 - V_1) = p\Delta V \longrightarrow \boxed{W_{gas} = p\Delta V}$$

What is Q?

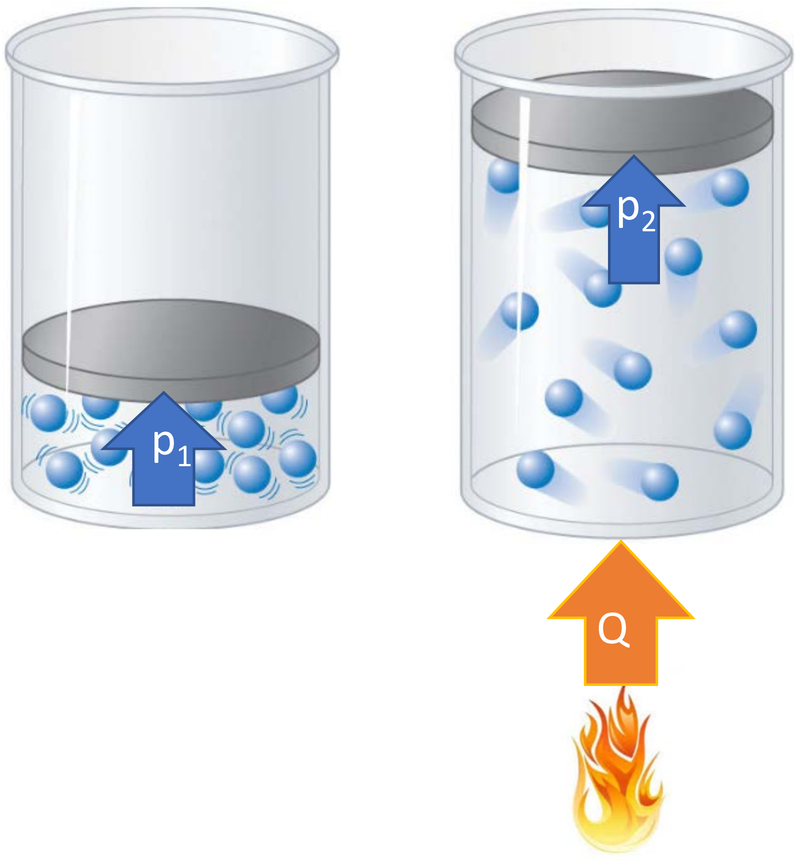
$$Q = \Delta E + W_{gas}$$
$$= \frac{f}{2} N k_B \Delta T + p\Delta V$$
$$= \left(\frac{f}{2} + 1\right) p\Delta V$$
$$= \left(\frac{f}{2} + 1\right) N k_B \Delta T$$

Now there is a nifty trick that's sometimes beneficial, to rewrite this expression in terms of other variables….

$$pV = N k_B T$$
$$\Delta(pV) = N k_B \Delta T$$
$$p\Delta V = N k_B \Delta T$$

F.7 Thermodynamic Processes

3. Isothermal process



Isothermal means constant temperature. Temperature can be constant if….

- so as gas's volume increases, and it does work, its temperature (and pressure) will drop. Heat must be added at rate sufficient to keep the temperature constant. This is usually done by putting the gas in contact with a large *thermal reservoir* at that same temperature.

What is W_{gas}?

$$W_{gas} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{Nk_BT}{V}\,dV = Nk_BT\int_{V_1}^{V_2}\frac{dV}{V} = Nk_BT\ln\left(\frac{V_2}{V_1}\right)$$

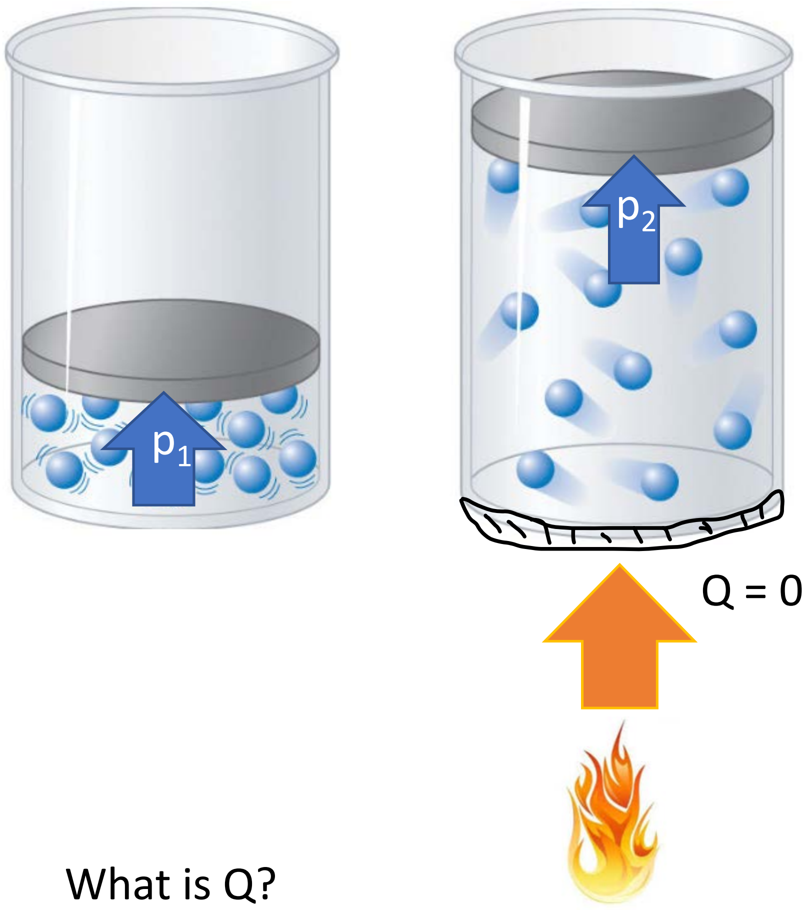
$$\boxed{W_{gas} = Nk_BT\ln\left(\frac{V_2}{V_1}\right)}$$

What is Q?

$$\begin{aligned} Q &= \Delta E + W_{gas} \\ &= \frac{f}{2}Nk_B\Delta T + Nk_BT\ln\left(\frac{V_2}{V_1}\right) \\ &= Nk_BT\ln\left(\frac{V_2}{V_1}\right) \end{aligned}$$

F.7 Thermodynamic Processes

4. Isentropic process



What is Q?

$$Q = 0$$

Isentropic means constant entropy. Entropy can be constant if….

- gas is insulated from heat sources/sinks, so Q = 0, or
- processes happens so fast that the gas has practically no chance to absorb or release heat.

What is W_{gas}?

$$W_{gas} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{Nk_BT}{V}\,dV$$

$$= \int_{V_1}^{V_2} \frac{const'}{V^\gamma}\,dV = \left.\frac{const'\cdot V^{1-\gamma}}{1-\gamma}\right|_{V_1}^{V_2}$$

$$= \frac{const'\cdot V_2^{1-\gamma}}{1-\gamma} - \frac{const'\cdot V_1^{\gamma}}{1-\gamma}$$

$$= \frac{p_2V_2^\gamma\cdot V_2^{1-\gamma}}{1-\gamma} - \frac{p_1V_1^\gamma\cdot V_1^{1-\gamma}}{1-\gamma}$$

$$= \frac{p_2V_2 - p_1V_1}{1-\gamma}$$

$$\boxed{W_{gas} = \frac{\Delta(pV)}{1-\gamma}}$$

Have problem because T and V both change during this process. Need to know how T changes with V, in order to integrate. Consider,

$$S = \text{doesn't change}$$

$$Nk_B \ln\left(\frac{T^{f/2}V}{\Phi N}\right) = \text{doesn't change}$$

$$\boxed{T^{f/2}V = const.}\ \boxed{= T_1^{f/2}V_1}\ \boxed{= T_2^{f/2}V_2}$$

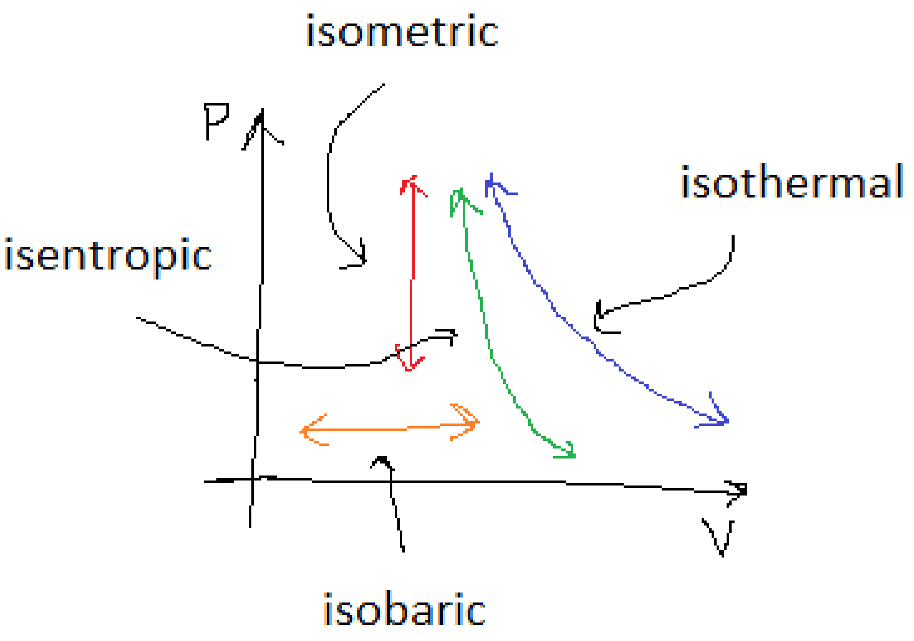
Another convenient way to write this is obtained by filling in T = pV/Nk$_B$. We get:

$$\boxed{pV^\gamma = const.'}\ \boxed{= p_1V_1^\gamma}\ \boxed{= p_2V_2^\gamma}$$

$$\boxed{\gamma = 1 + \frac{2}{f}}$$

F.7 Thermodynamic Processes

It is standard to draw these processes on a p-V diagram. And this is what they look like:



1. Isometric process

$$W_{gas} = 0 \qquad Q = \Delta E_{gas} + W_{gas}$$

2. Isobaric process

$$W_{gas} = p\Delta V \qquad Q = \Delta E_{gas} + W_{gas}$$

3. Isothermal process

$$W_{gas} = Nk_B T \ln\left(\frac{V_2}{V_1}\right) \qquad Q = \Delta E_{gas} + W_{gas}$$

4. Isentropic process

$$W_{gas} = \frac{\Delta(pV)}{1-\gamma} \qquad Q = \Delta E_{gas} + W_{gas}$$

F.7 Thermodynamic Processes

Say you have a hot-air balloon with present volume 10m^3, at standard temperature and pressure (T = 300K, p = 1atm = 101kPa). And you need to raise its volume to 15m^3 in order to have enough buoyancy to lift your crew. Presuming you add heat isobarically....

How much heat must you add?

$$Q = \Delta E_{gas} + W_{gas} = \frac{f}{2} N k_B \Delta T + p\Delta V = \frac{f}{2} p\Delta V + p\Delta V = \left(\frac{f}{2}+1\right) p\Delta V$$

$$Q = \left(\frac{5}{2}+1\right)(101\text{kPa})(15\text{m}^3 - 10\text{m}^3) = 1.77\text{MJ}$$

What will be the inside air's new temperature?

$$pV = Nk_B T$$

But don't know N....
A useful approach here is to set up a ratio between final and initial situations.

$$\frac{p_2 V_2}{p_1 V_1} = \frac{N k_B T_2}{N k_B T_1}$$

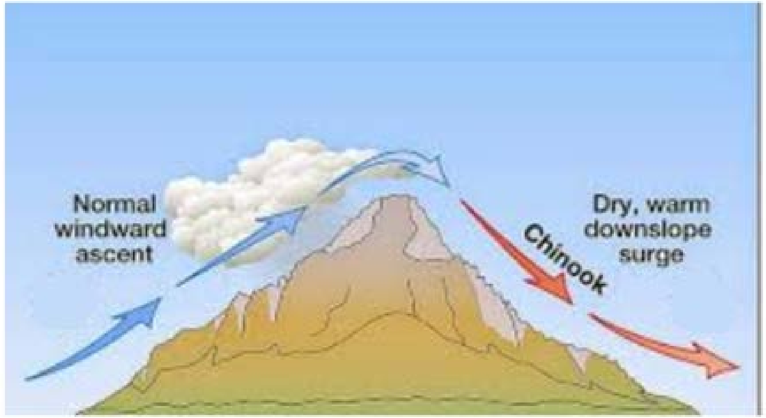
$$\frac{(101\text{kPa})(15\text{m}^3)}{(101\text{kPa})(10\text{m}^3)} = \frac{\cancel{Nk_B}\, T_2}{\cancel{Nk_B}\,(300\text{K})}$$

$$T_2 = (300\text{K})\left(\frac{15}{10}\right) = 450\text{K}$$

How much work will the expanding gas do on the balloon fabric?

$$W_{gas} = p\Delta V = (101\text{kPa})(15\text{m}^3 - 10\text{m}^3) = 505\,\text{kJ}$$

F.7 Thermodynamic Processes



Chinook winds are caused by moist winds convecting upwards from the sea up the slopes of a mountain. If the winds rise quickly up to the low pressure altitude, they will isentropically expand.

Why would an isentropic expansion cool the winds?

The winds do work against the surrounding atmosphere when they expand. This energy expenditure would cause them to lose temperature, since $\Delta E = (f/2)Nk_B\Delta T$. And if the expansion is isentropic, then there is no heat transferred to the winds from the outside air to make up for this lost energy.

The air precipitates moisture when it cools, and then descends down the slope of the mountain, recompressing isentropically and heating up as it does. Say the air goes from 0°C and 60kPa at the top of the mountain to 101kPa at the bottom. What is its new T?

$$T^{f/2}V = const. \qquad pV^{\gamma} = const.'$$

Neither of our equations directly relates pressure to temperature. But we can combine them…

$$T^{f/2}V = const. \rightarrow V = \frac{const.}{T^{f/2}}$$

$$pV^{\gamma} = const.' \rightarrow p\left(\frac{const.}{T^{f/2}}\right)^{\gamma} = const.'$$

$$\frac{p}{T^{f\gamma/2}} = const.''$$

$$\frac{p_1}{T_1^{f\gamma/2}} = \frac{p_2}{T_2^{f\gamma/2}}$$

$$\frac{60\text{kPa}}{(273\text{K})^{5(1+2/5)}} = \frac{101\text{kPa}}{T_2^{5(1+2/5)}}$$

$$\frac{60\text{kPa}}{(273\text{K})^{7}} = \frac{101\text{kPa}}{T_2^{7}}$$

$$T_2 = (273\text{K})\left(\frac{101\text{kPa}}{60\text{kPa}}\right)^{1/7}$$

$$T_2 = 294\text{K} = 22^{\circ}\text{C}$$